Modelling with R

Two Non-linear Regressions Examples

1. The data set in the file `rat_data.csv` gives the production of insulin in experimental animals, (rats), in response to a mixture of two drugs. The drug doses are in variables `x1` and `x1`, and the response, in suitable units, is in `y`. The data set comes originally from a paper by Darby and Ellis (1976), *Applied Statistics*, **25**: 298-299.

   A non-linear regression model of the following form has been suggested:

   $$y = \alpha + \delta \log\left(x_1 + \rho x_2 + \kappa\sqrt{\rho x_1 x_2}\right) + \epsilon, \qquad \epsilon \sim \mathrm{N}(0, \sigma^2)$$

   Some experimentation has suggested that suitable starting values could be

   $$\alpha = -17, \quad \delta = 10, \quad \rho = 0.05, \quad \kappa = -0.03$$

   - Fit the non-linear regression and examine the fit of the model.
   - Give the parameter estimates and their standard errors.
   - Fit the model in the alternative form

     $$y = \alpha + \delta \log\left(x_1 + \rho x_2 + \kappa\sqrt{x_1 x_2}\right) + \epsilon, \qquad \epsilon \sim \mathrm{N}(0, \sigma^2)$$

     and explain how the parameter estimates are related.

2. The data set in the file `bean_data.csv` relates to the growth of an experimental bean plants. The data originally comes from the book *Nonlinear regression modelling: a unified practical approach* (1983). Marcel Decker, by David A. Ratkowsky.

   The variable `y` gives the length of the plant after a fixed growth time and the variable `x` the amount of water supplied.

   Two possible non-linear regression models would be

   $$\begin{aligned} \text{Gompertz}: \quad y &= \alpha \exp\left(-\beta\gamma^x\right) + \epsilon \\ \text{Logistic}: \quad y &= \frac{\alpha}{1 + \exp\left\{-(x-\beta)/\gamma\right\}} \end{aligned}$$

   Self-starting regressin functions are available for both models, namely `SSgompertz` and `SSlogis` in the `stats` package, (which is attached by default).

   - Plot the data.
   - Fit both models and superimpose the fitted lines on the data points in two different colours.
   - Extend both models by adding a constant to the model function, say δ.
   - In each case, test the model that the additional constant improves the fit.
   - Add the two lines from the extended models to the plot, using the same colors as before, but a different line type.
   - Add a legend to your plot stating what the lines and symbols mean. Place the legend in the lower right-hand corner of the plot.